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RECURRING HARMONIC WALKS AND NETWORK MOTIFS IN WESTERN MUSIC

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Western harmony is comprised of sequences of chords, which obey grammatical rules. It is of interest to develop a compact representation of the harmonic movement of chord sequences. Here, we apply an approach from analysis of complex networks, known as "network motifs" to define repeating dynamical patterns in musical harmony. We describe each piece as a graph, where the nodes are chords and the directed edges connect chords which occur consecutively in the piece. We detect several patterns, each of which is a walk on this graph, which recur in diverse musical pieces from the Baroque to modern-day popular music. These patterns include cycles of three or four nodes, with up to two mutual edges (edges that point in both directions). Cliques and patterns with more than two mutual edges are rare. Some of these universal patterns of harmony are well known and correspond to basic principles of music theory such as hierarchy and directionality. This approach can be extended to search for recurring patterns in other musical components and to study other dynamical systems that can be represented as walks on graphs.

Keywords: Network motifs; music complexity; music perception; complex networks; design principles; graph theory.

1. Introduction

Tonal harmony is the harmonic system that has predominated classical music from the mid-17th century to the turn of the 20th century, as well as most popular Western music in the last century. Tonal music is characterized by a hierarchy of chords [6, 15, 39]. The *tonic*, which is the chord built on the first tone (marked I), is the most important, followed by the chords built on the fourth (the subdominant-IV) and fifth (dominant-V) tones. Musical harmony usually consists of a progression of chords obeying grammatical rules.

Classical theories of musical harmony [20, 29, 32, 33] have identified several widely used chord progressions in tonal harmony, and have characterized the probabilities of different types of chord transitions. More recently, several studies presented automated algorithms for the detection of patterns of harmony [3, 7, 9, 14, 16, 21, 22], mostly using string-matching algorithms. These approaches readily detected recurring patterns of chords. Several works have tried to capture the grammatical structure of tonal harmony by applying statistical tools used in natural language processing, such as N-grams and Markov models [26, 28, 36]. These statistical representations of harmonical patterns have found use in automatic retrieval of music and characterization of different musical styles. These methods usually rely on the identities of chords and their sequential relations.

Here, we seek to define a compact set of patterns which represent the basic dynamical movements of harmonic progression. Our method is based on a recent methodology for analyzing complex networks in terms of recurring building blocks termed *network motifs*. We describe a musical piece as a walk on a graph of chords, in which edges connect two chords appearing consecutively in a piece. We then detect the *dynamical patterns* of chord transitions — the subgraphs traced out by the piece as it traverses a variable length sequence composed of a set of chords. Unlike previous approaches, this method decouples the chord identities from the dynamical progression and focuses only on the type of dynamical pattern displayed by the chord transitions in the piece. We compare the dynamical patterns to patterns obtained in a set of randomized pieces and define the over-represented patterns as *dynamical motifs*. We find several highly significant recurring patterns that occur much more often in musical pieces than expected at random. Some of the patterns correspond to well-known progressions or to principles of composition such as directionality and hierarchy.

2. Network Motifs

Complex interaction networks [1, 2, 19, 25, 37] have recently been shown to contain recurring patterns that are highly significant compared to randomized networks [23, 24, 34]. These patterns are called network motifs. In biological regulation networks, in which nodes are proteins and edges represent the interactions between them, each of the motifs has been shown theoretically [17, 34] and experimentally [18, 30, 31, 40] to perform a specific information-processing function. Generally, different types of networks show different motifs, and networks can be classified into super-families according to their network motif profiles [23]. The motifs can be used as coarse-graining units to form a compact representation of the network [13]. Here, we ask whether this approach can be extended to detect meaningful patterns in musical harmony.

3. Dynamical Motifs in Musical Harmony

Each musical piece is represented by its chord sequence [Fig. 1(a)]. Major, minor, diminished, half-diminished and augmented chords are distinguished in the chord sequence whereas added notes and inversions are ignored (A and Am were regarded

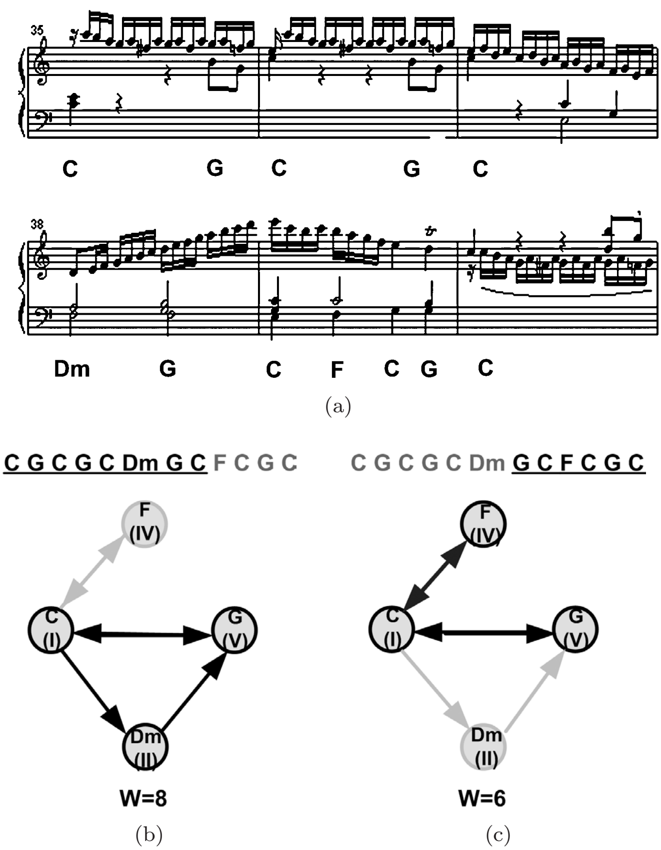
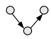
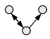
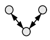
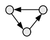
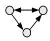
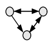
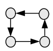
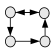
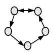
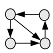
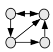
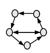
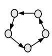


Fig. 1. (a) Chord sequence in a segment of Mozart sonata in A minor KV 310, movement I [38]. The chord sequence of this segment of the piece CGCGCDmGCFCGC includes two three-node patterns: (b) CGCGCDmGC and (c) GCFCGC. For (b) and (c), the chord sequences which form the three-node patterns are underlined and shown in black on the chord graph. Dynamical patterns were weighted (W) based on the length of their chord sequence.

as separate chords, while A, A6 and A/C were all considered A). The piece is then represented as a directed graph [Figs. 1(b) and (c)]. Each node represents a chord and a directed edge appears between chord X and chord Y if Y directly follows X in the chord sequence. The musical piece can thus be represented as a walk on this graph. We consider all sub-walks of this walk that visit n nodes. We do not count sub-walks which are included within a larger sub-walk with the same nodes. (In graph-theoretical terms, we enumerate all n-node edge-complete walks [12].) Each sub-walk traces out a pattern of nodes and edges that we term a *dynamical pattern* (see Table 1 for examples of 3–5 node dynamical patterns). Dynamical patterns are directed subgraphs that can be drawn, with the correct edge directionality, in a single motion without lifting the pen. There are eight possible types of topologically distinct 3-node dynamical patterns, 109 4-node dynamical patterns, 5,702 5-node dynamical patterns, etc. Each dynamical pattern is assigned a weight that is equal

Table 1. Consensus motifs and their frequency (numbers and percents of pieces in which they are motifs) in the analyzed database. Shown are all the three-node consensus motifs, as well as 4-node and 5-node consensus motifs with no dangling edges. Patterns 4 and 7 are motifs in many 20th century popular pieces, but are not found as motifs in classical pieces.

Pattern #	Pattern	Percent of classical pieces (75)	Number of 20th century popular pieces (484)	Percent of all pieces (559)
1		4 (5%)	36 (7%)	40 (7%)
2		8 (11%)	31 (6%)	39 (7%)
3		4 (5%)	36 (7%)	40 (7%)
4		1 (1%)	51 (11%)	52 (9%)
5		4 (5%)	44 (9%)	48 (9%)
6		7 (9%)	53 (11%)	60 (11%)
7		1 (1%)	47 (10%)	48 (9%)
8		13 (17%)	30 (6%)	43 (8%)
9		3 (4%)	8 (2%)	11 (2%)
10		6 (9%)	27 (6%)	33 (6%)
11		3 (4%)	9 (2%)	12 (2%)
12		3 (4%)	10 (2%)	13 (2%)
13		2 (3%)	11 (2%)	13 (2%)

to the total length of the walks that the musical piece performs along the pattern [Figs. 1(b) and (c)].

The present method detects patterns of variable length with the same inherent topology. For example, CGC, CGCG, CGCGC, etc., where CGC represents the chord sequence C major → G major → C major, all share the dynamical pattern $X \longleftrightarrow Y$. The method thus focuses on the pattern rather then on the identity of each node. For example, both CFGC and CGFC are realizations of the same dynamical pattern $X \rightarrow Y \rightarrow Z \rightarrow X$. Each pattern is further given a weight, defined as the length of the chord sequence that corresponds to the pattern. For example, the three short chord sequences CGC, CGCG, CGCGC all correspond to the pattern $X \longleftrightarrow Y$ with weights $3, 4, 5$, respectively.

Dynamical patterns and weights can be readily detected within musical pieces. Consider the example in Fig. 1, a section from the first movement of Mozart's sonata in A minor KV310. The annotated chord sequence is:

C, G, C, G, C, Dm, G, C, F, C, G, C. This piece contains two dynamical patterns made of three chords (nodes). The first dynamical pattern corresponds to the chord sequence C, G, C, G, C, Dm, G, C. The pattern traced out by this sequence is a 3-node cycle with one mutual edge [Fig. 1(b), pattern 5 in Table 1]. This dynamical pattern is assigned a weight of 8, the length of the chord sequence tracing it. The second dynamical pattern is composed of the chord sequence G, C, F, C, G, C. The pattern traced out by this sequence is a chain of two mutual edges [Fig. 1(c), pattern 3 in Table 1], and is assigned a weight of 6.

To detect which dynamical patterns are "significant", we compared each piece to an ensemble of randomized pieces, composed of random permutations of the chord sequence of the entire piece. In both the real and in each randomized piece, each dynamical pattern was assigned a total weight which is the sum of the weights of all its appearances in the piece. The statistical significance of a pattern was measured using a Z-score:

$$Z_i = \frac{W^i_{\text{real}} - W^i_{\text{rand}}}{\sigma^i_{\text{rand}}},$$

where W^i_{real} is the total weight of dynamical pattern i in the real piece, and W^i_{rand} and σ^i_{rand} are the mean and standard deviation of the total weights of dynamical pattern i in the randomized pieces. *Dynamical motifs* are those dynamical patterns that occur in the real piece much more often than in the randomized ensemble (Z-score threshold was adjusted using a Bonferroni correction method for multiple hypotheses testing: Z-score > 2.5 for three-node patterns, Z-score > 3 for four-node patterns, and Z-score > 4 for five-node patterns).

We assembled a database of 559 pieces, including 75 classical pieces and 484 popular 20th century pieces for which chord annotation was available. The classical pieces include compositions from the Baroque to the Romantic eras [4, 5, 10, 11, 27, 38]. The popular pieces [41] encompass works by the Beatles (130 songs), Rolling Stones (119 songs), Bob Dylan (118 songs), Simon and Garfunkel (21 songs), Elvis Presley (23 songs), Willie Nelson (30 songs) and Johnny Cash (43 songs). We searched for motifs of size 3–5 nodes.

We found that most of the pieces (60%) displayed a small set of highly significant dynamical motifs. We then sought to find motifs that are shared by many pieces. We term such motifs *consensus motifs* (Table 1 and Fig. 3). To define consensus motifs, we considered the number of pieces in which dynamical pattern i appears as a motif as a random variable X_i. To create a distribution $P(X_i)$ for this variable, we constructed 50 databases of randomized pieces. Each database consisted of 559 pieces, where each piece was a random permutation of the corresponding original piece. We then searched for motifs in all pieces in each database. A dynamical pattern was considered a consensus motif if: $P(X_i > N_i) < 0.05$, where N_i is the number of pieces in which the pattern appeared as a motif in the database of real (non-randomized) pieces.

4. Musical Meaning of Consensus Motifs

We found several highly significant consensus motifs, shared by many pieces throughout all periods. The consensus motifs of size 3–5 are shown in Table 1. Notable motifs include cycles (patterns $4, 7$ in Table 1) and cycles with one mutual edge (patterns $5, 8, 9, 11, 12$ in Table 1). Two three-node patterns, the clique ($X \longleftrightarrow Y$, $X \longleftrightarrow Z$, $Y \longleftrightarrow Z$) and a mutual edge preceded by an edge ($X \rightarrow Y$, $Y \longleftrightarrow Z$) are not consensus motifs.

After identifying each consensus motif, we analyzed which chords take part in the motif. One of the most significant consensus motifs (48/559 pieces) consists of a three-chord cycle involving one mutual edge $X \longleftrightarrow Y$, $Y \rightarrow Z$, $Z \rightarrow X$ (pattern 5, Table 1, Figs. 1(b) and 2). In classical pieces, the mutual edge in this motif often represents a I–V–I transition, followed by a I–II–V–I cycle (as in Mozart sonata in A minor KV310, Fig. 1) or a I–IV–V–I cycle. In 20th century popular pieces, this motif often represents the 12-bar blues progression: I($\times$4)–IV($\times$2)–I($\times$2)–V–IV–I($\times$2) (Fig. 2). Examples for this motif include Elvis' "Blue suede shoes" and Bill Haley's "Rock around the clock" (Fig. 2).

Generalizations of this pattern to a larger number of nodes are also common motifs throughout the music we analyzed. For example, four- and five-node cycles with one mutual edge (the mutual edge often corresponding to a I–V–I or I–IV–I progression) are both consensus motifs (Table 1, patterns 8 and 9, Fig. 3, squares). The mutual edge in these motifs usually acts to establish the tonic (I) as the progression's goal, through its relationship to the dominant V or subdominant IV chord [38]. Pattern 8 appears in 11/13 classical pieces as the progression: I–V–I–VI–II–V–I, whereas in popular pieces the chord progressions of this pattern are more varied.

An additional significant consensus motif (pattern 3 of Table 1, 40/559 pieces) is a chain of two mutual edges. This often involves a I–V–I–IV–I progression. Thus, it includes the three most important chords in the tonal hierarchy, and has a pleasing symmetry due to the fact that it has equal tonal spacing of a fifth between both

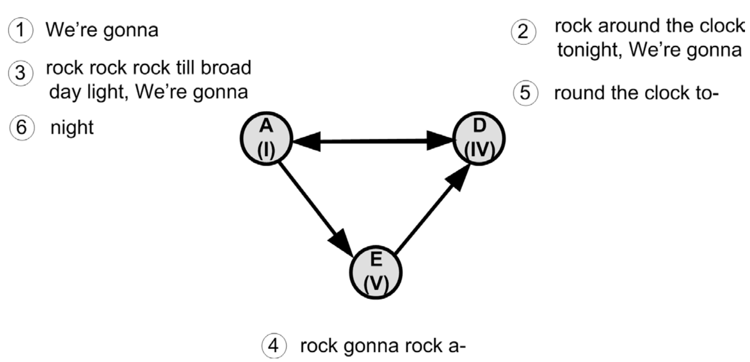


Fig. 2. An example of the 12-bar blues motif in the song "Rock around the clock" by Bill Haley. Circled numbers indicate consecutive lyric lines.

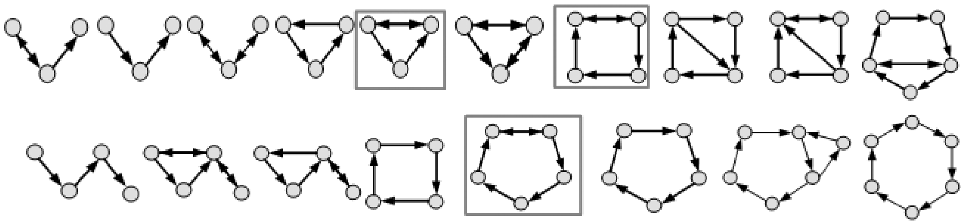


Fig. 3. Consensus motifs found in classical and 20th century popular pieces. Squares mark patterns made of a single cycle with one mutual edge. Five-node consensus motifs with dangling single edges are not shown.

I–V and IV–I. An additional pattern, with two mutual edges and a cycle, is also a consensus motif (pattern 6, 60/559 pieces).

Chains of a mutual edge followed or preceded by a single edge [patterns 2 and 7 in Fig. 4(b)] commonly occur, usually as parts of larger 4–5 node cycles. Notably, however, only pattern 2 was a consensus motif. This asymmetry is due to appearances of pattern 2 at the beginning of a piece, where a I–V–I harmonic progression is often used to establish the tonic. On the other hand, pieces ending with a I–V–I cadence usually involve three-node cycles such as patterns 5 and 6. Additional consensus motifs include combinations of three-, four- and five-node cycles and cycles with one mutual edge (Table 1, patterns 10–12).

We find that dynamical patterns with more than two mutual edges are very rarely encountered as motifs, and in fact are often anti-motifs (occur significantly less often than at random). For example, the three-node fully connected clique ($X \longleftrightarrow Y$, $X \longleftrightarrow Z$, $Y \longleftrightarrow Z$) appeared as a motif in only 14/559 pieces, and was a strong anti-motif in many pieces. This might be related to the fact that more than two mutual edges can obscure the central role of the tonic and the directionality of the piece [8, 38].

The most prominent difference between the motifs of classical and 20th century popular pieces in our database was in three- and four-node cycles (patterns 4 and 7 in Table 1), which appeared frequently in 20th century popular pieces (51/484 and 47/484) but were motifs in only one of the classical pieces.

5. Classification of Musical Pieces

We sought to compare different pieces based on their motifs. We employed a method that compares motif profiles and is insensitive to the sizes of the musical pieces [23]. We calculated the significance profile (S) of the appearances of the eight possible three-node dynamical patterns (Fig. 4). The significance profile is defined as the normalized vector of Z-scores of each pattern:

$$S_i = \frac{Z_i}{\sqrt{\sum_{i=1}^{8} Z_i^2}},$$

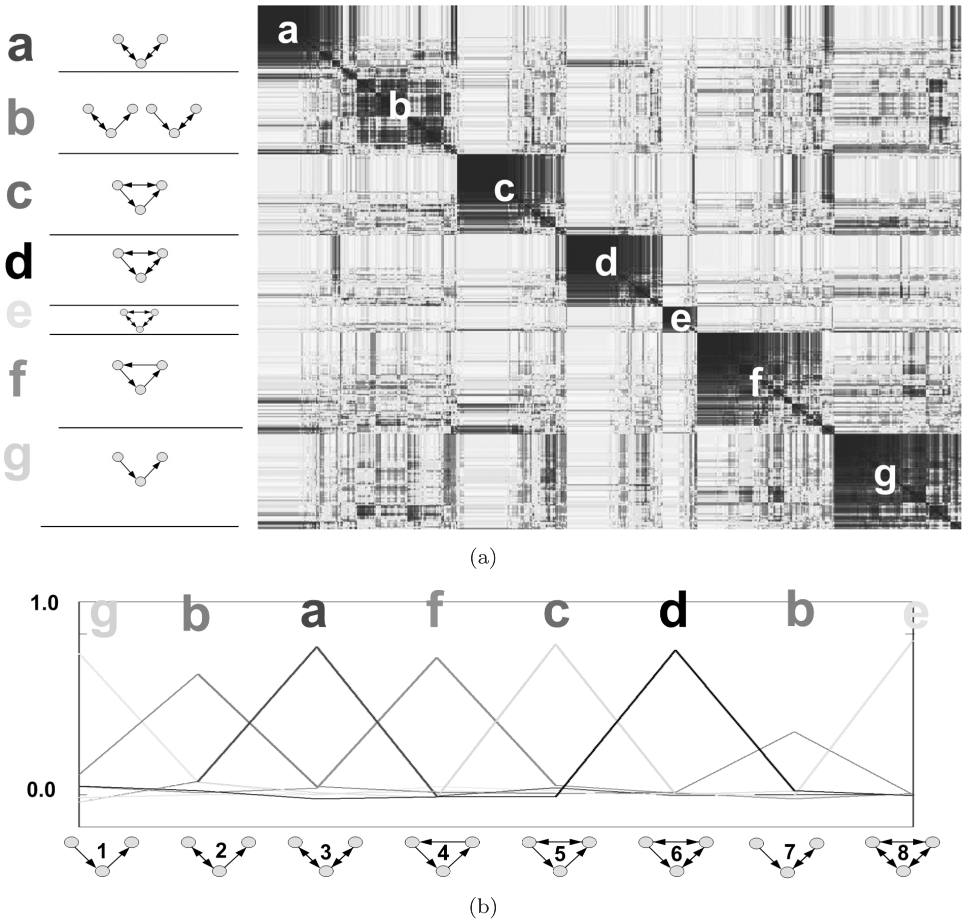


Fig. 4. (a) Correlation coefficient matrix of the subgraph significance profiles of 559 musical pieces. The pieces were clustered according to significance profiles similarity (using hierarchical clustering of Matlab 7). The main three-node motifs in each cluster are shown on the left. (b) Average significance profiles of each of the seven clusters, a–g, in (a), showing the main three-node motif in each cluster.

where Z_i is the Z-score of pattern i ($i = 1, \ldots, 8$). We find that musical pieces can be classified into seven families based on the similarity in their significance profiles for three-node patterns (Fig. 4). Most families are characterized predominantly by one three-node pattern [Fig. 4(b)]. One of the families (family b) is characterized by two three-node patterns, pattern 2 and 7. Individual pieces rarely have two or three different three-node motifs, but cluster into one of the families based on their most significant motif.

What is the meaning of the classification of pieces according to their dynamical patterns? One might hypothesize that each dynamical pattern can have a cognitive correlate and may convey a distinct feeling such as "static" (patterns with

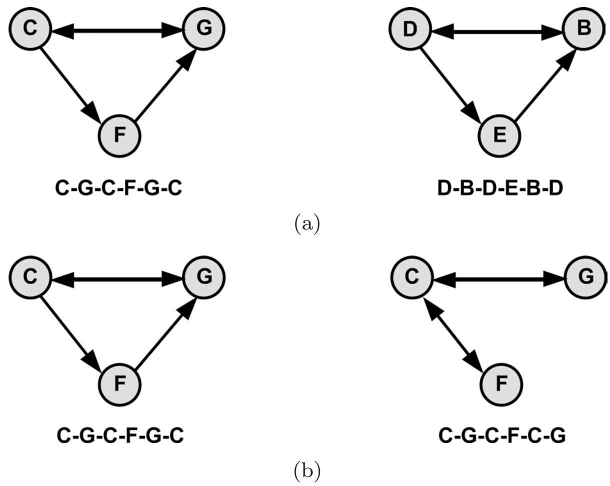


Fig. 5. Chord sequences representing (a) the same dynamical pattern and different chord identities and (b) the same chord identities and different dynamical pattern.

mutual edges) or "non-static" (cycle patterns). It will be interesting to study the cognitive effect of short chord sequences representing different dynamical patterns, using psychophysical experiments. One possible experiment is to study the cognitive responses to different dynamical patterns with the same chords and to the same dynamical patterns with different chords (Fig. 5). This type of experimental design can help separate the effects of chord identity and dynamical pattern on music perception.

The present approach considers only dynamical patterns, and obviously does not relate to most of the beauty and essence of musical harmony, which is borne by the chord identities and their interrelations. Rather, this approach aims at presenting a graphical vocabulary of harmonic patterns in a piece, in a systematic manner which does not depend on exact chord identities, or chord-sequence length.

The present approach is not applicable to music which is not based on harmonic variation, such as some forms of non-Western music. It is also difficult to analyze some modern and atonal music, in which the abundance of enharmonic chords and the avoidance of a single tonal center prohibit unequivocal chord annotation.

We presented an approach to systematically detect significant dynamical patterns which are walks on graphs representing transitions between states of a dynamic system, and applied it to networks of harmony in music. This method allows the detection of variable length patterns which share the same basic topology. Only a tiny fraction of the possible patterns are actually found as motifs. It is interesting that music of different eras and styles converge on a relatively small set of patterns. These patterns are presumably favored due to the cognitive effects they produce in the listener [6, 8, 15, 39]. The recurring patterns, which predominantly include cycles and cycles with a mutual edge often involving the tonic, corresponded to well-studied principles of tonal harmony. Cliques and patterns with more than two mutual edges were rare. This may be related to the preference for directionality in music [8, 38], and for a single "center of gravity" represented by the tonic.

The coarse-grained dynamical pattern representation of the harmonic movement may be used for additional feature analysis, such as musical complexity [35]. The present approach can in principle be applied to analyze recurring patterns in other musical components such as melody and rhythm. It may also help to study the dynamics of other systems that move through discrete states (e.g. states of robotic arms or states of computer programs during runs).

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